Collective diffusion and a random energy landscape

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(Received 24 February 2000)

Starting from a master equation in a quantum Hamiltonian form and a coupling to a heat bath, we derive an evolution equation for a collective hopping process under the influence of a stochastic energy landscape. Different equations result for an arbitrary occupation number per lattice site or in a system under exclusion. Based on scaling arguments it will be demonstrated that both systems belong below the critical dimension d_c to the same universality class, leading to anomalous diffusion in the long time limit. The dynamical exponent z can be calculated by an $\epsilon = d_c - d$ expansion. Above the critical dimension we discuss the differences in the diffusion constant for sufficient high temperatures. For a random potential we find a higher mobility for systems with exclusion.

PACS number(s): 05.70.Ln, 05.50.+q, 75.10.Hk, 82.20.Mj

I. INTRODUCTION

Many systems behave on the phenomenological level essentially randomly and therefore other approaches for the theoretical treatment have to be employed. The randomness, resulting from stochastic forces or intrinsic in the underlying microscopic theory, inevitably leads to the description of such systems in terms of probabilities and expectation values [1,2]. The time development of probability is usually found using a master equation. The past years have seen an exciting new development based on the observation [3] of the close relationship between the Markov generator of the master equation and a time evolution operator acting on a many-particle Fock space [4,5]; for some recent reviews see Refs. [6,7]. The new insight has led to a series of remarkable exact solutions for the stochastic dynamics of interacting particle systems, for a recent overview see Ref. [8]. Despite the exact results, the mentioned method has also been fruitful in deriving an approximate description of other models such as the facilitated kinetic Ising system as a candidate for glassy systems [9–11] or in branching and annihilation random walks [12]. Whereas Ref. [3] (see also Ref. [13]) is concerned with a mapping of the master equation to a representation in terms of second-quantized bosonic operators, great progress for exactly solvable models had been achieved by mapping to spin-1/2 Pauli operators [8]. This mapping to spin systems applies to processes where each lattice site can be occupied by only a finite number of particles. Physically, this restriction may be hardcore constraints or fast on-site annihilation processes. Obviously, such a mapping simulates the exclusion principle for classical lattice models within cellular automata.

In the present paper the Fock-space description is applied for systems far from equilibrium which are coupled to a heat bath. In particular, we discuss the collective hopping process of a classical many body system, coupled to the mentioned heat bath, and under the influence of a random energy landscape realized by a stochastic activation energy. The particles making random walks have to overcome spatially distributed energy barriers. As a consequence, the hopping process is accomplished by a competing force field which can give rise to anomalous diffusion. Further, the analysis for both cases, the bosonic and the exclusive one, should be different. In the first case the particles should find more rapidly the local energy minima. However, because of that their mobility could be reduced. As a consequence of the random walk where the particles have to overcome spatially distributed energy barriers, the resulting effective force field can give rise to anomalous diffusion. It is well known that one of the reasons for anomalous diffusive behavior can be traced back to the influence of a stochastic force field below a critical dimension [14,15]. An alternative way of achieving self-induced anomalous diffusion had been discussed recently [16] introducing a feedback coupling between the diffusive particle and its local environment. Both the disorder and the memory controlled feedback may lead to a subdiffusive behavior or to localization. The analytical approach [16] could be confirmed by simulations in one and two dimensions where at the critical dimension $d_c = 2$ logarithmic corrections in the mean square displacement have been found [17,18].

Here, we demonstrate that the Fock space approach leads in both cases, bosonic and under exclusion, to anomalous diffusion. Within the long time limit and on a large spatial scale both systems belong to the same universality class.

II. QUANTUM APPROACH TO NONEQUILIBRIUM

Before we present a more precise definition of our model in terms of a master equation, let us describe the situation in mind verbally. We consider the time evolution of particle distributions on a regular lattice where each lattice site is related to a well defined local energy ε_i. The complete energy landscape is defined by the set of randomly distributed energies ε_i. Particles are updated asynchronously at a rate μ. The update corresponds to choosing a neighboring site and moving the particle to it with a probability that depends on the random potential via a Boltzmann factor. To be more specific, a particle hops from site j to a neigboring point i subject to the condition that the sites j and i are characterized by the mentioned random energy which the hopping particle has to overcome. Additionally, the process is strongly influenced by the occupation number of the lattice sites. Whereas in the Bose case an arbitrary occupation number of each lattice site is allowed, all hopping attempts which would lead to a double occupancy of a site are rejected in the model with

exclusion (hardcore repulsion).

To include both conditions, the local energy distribution as well as the occupation rules per site, our analysis is based on a master equation

$$\partial_t P(\vec{n},t)=L' P(\vec{n},t), \tag{1}$$

where $P(\vec{n},t)$ is the probability that a certain configuration characterized by a state vector $\vec{n}=(n_1,n_2,\ldots,n_N)$ is realized at time t. There are two special cases: either each lattice site is occupied by an arbitrary number of particles $n_i=0,1,2,\ldots$, or as in a lattice gas $n_i=0,1$. Further, the occupation numbers n_i are considered as the eigenvalues of the particle number operator defined by creation operators $d_i^\dagger$ or by annihilation operators d_i. The problem is to formulate the dynamics in such a way that the possible realizations for the occupation numbers are taken into account explicitly. The situation in mind can be analyzed in a seemingly compact form using the master equation in a quantum Hamilton formalism [3,4,2,5,19,20]; for recent reviews see Refs. [7,8]. The dynamics is determined completely by the form of the evolution operator L', specified below, and the commutation relations of the underlying operators $d_i^\dagger$ and d_i. Within that approach [3] the probability distribution $P(\vec{n},t)$ is related to a state vector $|F(t)\rangle$ in Fock space according to $P(\vec{n},t)=\langle\vec{n}|F(t)\rangle$. The basic vectors $|\vec{n}\rangle$ are composed of the operators $d_i^\dagger$ and d_i. Using the relation

$$|F(t)\rangle=\sum_{n_i} P(\vec{n},t)|\vec{n}\rangle \tag{2}$$

the master Eq. (1) can be transformed into an equivalent one in Fock space,

$$\partial_t|F(t)\rangle=L|F(t)\rangle, \tag{3}$$

where the operator L' in Eq. (1) is mapped onto the operator $L=\Sigma|\vec{m}\rangle L'_{mn}\langle\vec{n}|$ in Eq. (3). It should be emphasized that the procedure is up to now independent of the realization of the basic vectors. Originally, the method had been applied for the Bose case [3,4,13]. Recently, an extension to restricted occupation numbers (two discrete orientations) was proposed [2,5,19]. Further extensions to p-fold occupation numbers [11] as well as to models with kinetic constraints [21] and to systems with two heat baths [22] are possible.

As shown by Doi [3], the average of an arbitrary physical quantity $B(\vec{n})$ can be calculated by the average of the corresponding operator $B(t)$,

$$\langle B(t)\rangle=\sum_{n_i} P(\vec{n},t)B(\vec{n})=\langle s|B|F(t)\rangle, \tag{4}$$

with the state function $\langle s|=\Sigma\langle\vec{n}|$. The evolution equation for an operator $B(t)$ now reads

$$\partial_t\langle B\rangle=\langle s|[B(t),L]|F(t)\rangle. \tag{5}$$

As a result of the procedure, all the dynamical equations governed by the classical problem are determined by the structure of the evolution operator L and the commutation rules of the operators.

III. COUPLING TO A HEAT BATH

The evolution operator for a collective hopping process is different for an arbitrary occupation number, denoted as the Bose case, or a restricted occupation number, denoted as the Fermi case. For the last system the operator L_f reads [9]

$$L_f=\mu\sum_{\langle i,j\rangle}[d_i^\dagger d_j-(1-n_i)n_j], \tag{6}$$

where μ is the hopping rate between adjacent sites i and j. The occupation number operator $n_i=d_i^\dagger d_i$ is related to the spin operator by the relation $S_i=1-2n_i$ and the commutation rule is $[d_i,d_j]=\delta_{ij}(1-2n_i)$.

For the Bose case we get

$$L_b=\mu\sum_{\langle i,j\rangle}(d_i^\dagger d_j-n_j), \tag{7}$$

where $d_i^\dagger$ and d_i fulfill the Bose commutation rules. A generalization to processes under the coupling to a heat bath with a fixed temperature T is discussed in Ref. [9]. As demonstrated in Refs. [9,23] the evolution operator has to be replaced by

$$L_f=\mu\sum_{\langle i,j\rangle}[(1-d_id_j^\dagger)\exp(-\beta H/2)d_i^\dagger d_j\exp(\beta H/2)], \tag{8}$$

where the hopping rate μ defines a microscopic time scale, $\beta=T^{-1}$ is the inverse temperature of the heat bath, and H is the Hamiltonian as a measure of energy. A further generalization, realized by introducing different local heat baths, is discussed in Ref. [22]. In the bosonic case the generalization to finite temperatures leads to

$$L_b=\mu\sum_{<i,j>}[(1-\delta_{ij})\exp(-\beta H/2)d_i^\dagger d_j\exp(\beta H/2)]. \tag{9}$$

Here we study the case in which the Hamiltonian H in Eqs. (8),(9) is simply given by a stochastic energy landscape defined by the energy functional

$$H=\sum_i \varepsilon_i n_i. \tag{10}$$

Whenever the energy is positive the empty site is energetically favored. Further, ε is assumed to be a stochastic local energy the distribution of which will be introduced below based on the continuous representation. In this manner, the model describes a collective hopping process where the jumping particles are subjected to a local random energy ε_i which supports or prevents the hopping process with a probability proportional to $\exp(\pm\varepsilon_i/2T)$. Taking into account the commutation rules, we get in both cases

$$e^{-\beta H/2} d_i e^{\beta H/2} = d_i e^{\varepsilon_i/2T}, \quad e^{-\beta H/2'} d_i^\dagger e^{\beta H/2} = d_i^\dagger e^{-\varepsilon_i/2T}. \tag{11}$$

Using Eq. (5) and the algebraic properties of Pauli operators, the evolution equation for the averaged density reads

$$\mu^{-1}\partial_t\langle n_r\rangle = \sum_{j(r)} \left\{ \exp[(\varepsilon_j-\varepsilon_r)/2T]\langle n_j\rangle - \exp[(\varepsilon_r-\varepsilon_j)/2T] \times\langle n_r\rangle - 2\sinh\left(\frac{\varepsilon_j-\varepsilon_r}{2T}\right)\langle n_r n_j\rangle \right\} \tag{12}$$

In the Bose case the evolution equation is much simpler:

$$\mu^{-1}\partial_t\langle n_r\rangle = \sum_{j(r)} \{\exp[(\varepsilon_j-\varepsilon_r)/2T]\langle n_j\rangle - \exp[(\varepsilon_r-\varepsilon_j)/2T]\langle n_r\rangle\}. \tag{13}$$

Both equations reflect the conservation of the particle number which will be more apparent in a continuum representation. In the special case of a constant energy $\varepsilon_r=\varepsilon_j$ the conventional diffusion equation results in a discrete version. When the energy changes from site to site the nonlinear Eq. (12) is the first step in a whole hierarchy of evolution equations. Assuming now smoothly changing energy ε_r and density n_r a gradient expansion is appropriate up to the order l^2, where l is the lattice size. To make the expansion invariant under the underlying rotational symmetry we have to use the following identity:

$$\sum_{j(r)} \exp[(\varepsilon_j-\varepsilon_r)/2T]\langle n_j\rangle = \sum_{j(r)} \langle n_r\rangle + \exp(-\varepsilon_r/2T)\sum_{j(r)} [\exp(\varepsilon_j/2T)\langle n_j\rangle - \exp(\varepsilon_r/2T)\langle n_r\rangle]. \tag{14}$$

Such an expression reads, in a continuous representation including terms of the order l^2,

$$z n(\mathbf{r},t) + \exp(-\varepsilon(\mathbf{r})/2T]\nabla^2\{\exp[\varepsilon(\mathbf{r})/2T]n(\mathbf{r},t)\}$$

with the averaged density $\langle n_r\rangle \equiv n(\mathbf{r},t)$; z is the number of nearest neighbors. After decoupling the nonlinear term in Eq. (12) and performing the continuous limit the density $n(\mathbf{r},t)$ obeys the following nonlinear diffusionlike equation:

$$\mu^{-1}l^{-2}\partial_t n = \nabla^2 n + n(1-n)\frac{\nabla^2\varepsilon}{T} + (1-2n)\boldsymbol{\nabla} n\cdot\boldsymbol{\nabla}\varepsilon/T. \tag{15}$$

In a system with exclusion the density couples in a nonlinear manner to the stochastic energy field $\varepsilon(\mathbf{r})$. Due to the exchange coupling of the evolution operator L in Eq. (6) the resulting equation (15) is a conserving one where the current is given by

$$\mathbf{j}_f = -\boldsymbol{\nabla} n - n(1-n)\frac{\boldsymbol{\nabla}\varepsilon}{T}. \tag{16}$$

In the Bose case we find after performing the continuous limit that the density $n(\mathbf{r},t)$ obeys the following exact equation:

$$\mu^{-1}l^{-2}\partial_t n = \nabla^2 n + \frac{1}{T}\boldsymbol{\nabla}[n\boldsymbol{\nabla}\varepsilon]. \tag{17}$$

The conservation law is manifested in the current

$$\mathbf{j}_b = -\boldsymbol{\nabla} n - n\frac{\boldsymbol{\nabla}\varepsilon}{T}. \tag{18}$$

The resulting equation is no more than the conventional diffusion equation under an additional drift term where the Einstein relation is automatically fulfilled. Notice that one can derive a similar equation when the system is coupled to two heat baths with different temperatures. In that case one has to replace $\varepsilon(\mathbf{r})/T$ by $\nu/T(\mathbf{r})$, where ν is the chemical potential and $T(\mathbf{r})$ is the local temperature, see also Ref. [22]. In the Bose case Eq. (17) depends on the density in a linear manner. It is of Fokker-Planck type when the density $n(\mathbf{r},t)$ is considered as the single probability distribution to find a particle at site $\mathbf{r}$ at time t. Such an interpretation is always possible because we have not taken into account any interactions. Therefore, the particles are independent of each other and the concentration field behaves as the probability distribution of a single particle of this system. In contrast to the case of an arbitrary occupation, the current $\mathbf{j}_f$ includes a term $n(1-n)$ which is characteristic of systems with exclusion. Due to the exclusion principle, the systems reveal a kind of correlation which leads even in the mean field limit to a nonlinear current. Following the discussion for the Bose case Eq. (15) can be interpreted as a nonlinear Fokker-Planck equation for a single particle. The nonlinearity reflects the feedback of a particle to itself due to the excluded volume effect.

It seems to be more appropriate to introduce the force vector $\mathbf{f}(\mathbf{r}) = -\nabla\varepsilon(\mathbf{r})$. The evolution equation in the Bose case now reads

$$\mu^{-1}l^{-2}\partial n(\mathbf{r},t) = \nabla^2 n - \frac{1}{T}\mathbf{f}\cdot\boldsymbol{\nabla} n - \frac{1}{T}\boldsymbol{\nabla}\cdot\mathbf{f} n. \tag{19}$$

In the Fermi case the corresponding equation is

$$\mu^{-1}l^{-2}\partial n(\mathbf{r},t) = \nabla^2 n - \frac{1}{T}\mathbf{f}\cdot\boldsymbol{\nabla} n(1-2n) - \frac{1}{T}n(1-n)\boldsymbol{\nabla}\cdot\mathbf{f}. \tag{20}$$

When the force field $\mathbf{f}(\mathbf{r})$ is a stochastic one the system offers anomalous diffusive behavior [14,15].

IV. SCALING

Now let us discuss both equations when the force field is a stochastic pure spatial-dependent field, the correlator of which is given by

$$\overline{f_\alpha(\mathbf{r})f(\mathbf{r}')} = \phi_{\alpha\gamma}(\mathbf{r}-\mathbf{r}'), \quad \overline{f_\alpha(\mathbf{r})} = 0. \tag{21}$$

After averaging over the distribution function of the force field the system is homogeneous, depending only on the dif-

ference between the spatial coordinates. The most general form of the function $\phi_{\alpha\gamma}$ is given in a Fourier representation by

$$\phi_{\alpha\gamma}=A(\vec{q})(\delta_{\alpha\gamma}-n_\alpha n_\gamma)+B(\vec{q})n_\alpha n_\gamma \quad \text{with } n_\alpha=\frac{q_\alpha}{q}. \tag{22}$$

Introducing dimensionless variables $x\to x\Lambda^{-1}$, $t\to t\Lambda^{-z}$, where z is the dynamical critical exponent and, further, $n\to n\Lambda^d$ and, according to Eq. (22), for constant A and B $\mathbf{f}\to\mathbf{f}\Lambda^{d/2}$, we find the critical dimensionality $d_c=2$. For $d\leqslant 2$ the term proportional to $\nabla(\mathbf{f}n)$ is relevant, whereas the additional term in the case of exclusive motion $\propto n\mathbf{f}\nabla n$ is only relevant for $d<2/3$. That means for the physical dimension $d\geqslant 1$ both models belong to the same universality class, where only $d\leqslant 2$ the disorder is relevant. Physically the result is obvious because in the long time limit and for a large spatial scale the Fermi system can be considered to consist of blocks of increasing size. The larger such a block is the more irrelevant it is to distinguish both cases, arbitrary occupation and restricted occupation. In the case where $d\leqslant 2$, the system reveals anomalous diffusive behavior as has been demonstrated for a similar model, not for the density $n(\mathbf{r},t)$ but for the probability P of finding a particle at time t at point $\mathbf{r}$. Making the same calculation we end up with the flow equations for the dimensionless coupling parameters $D=\mu l^2$, $a=(A/D^2T^2)K_d$, $b=(B/D^2T^2)K_d$, with $K_d(2\pi)^d$: the volume of the d-dimensional unit sphere and $\epsilon=2-d$, $\xi=\ln(\Lambda_0/\Lambda)$,

$$\frac{\partial D}{\partial\xi}=D\left[z-2+\frac{a(d-1)}{d}-\frac{b}{d}\right],$$

$$\frac{\partial a}{\partial\xi}=a\left[\epsilon-a+\frac{b(d-1)}{d}\right],$$

$$\frac{\partial b}{\partial\xi}=b\left[\epsilon-\frac{a}{d}\right]. \tag{23}$$

In the same manner one can derive an equation for the mean square displacement $R=\Lambda^2 s(D,a,b)$ with $s=\langle\mathbf{r}^2\rangle$. The flow equation can be written as

$$2s=\frac{\partial s}{\partial D}\partial_\xi D+\frac{\partial s}{\partial a}\partial_\xi a+\frac{\partial s}{\partial b}\partial_\xi b. \tag{24}$$

That equation leads to a scaling behavior of the mean square displacement in the vicinity of the fixed points of Eqs. (23). In order to keep the diffusivity D fixed at its bare value, the effective dynamical exponent $z(\xi)$ satisfies $z(\xi)=2+b(\xi)/d+a(\xi)(1-d)/d$. When the disorder is irrelevant, the fixed points are $a^\star=b^\star=0$ and the exponent is $z=2$. For the fixed point $a^\star=\varepsilon d$, $b^\star=0$, $z=2-\varepsilon$ results and for $a^\star=b^\star=\epsilon d$ we find $z=2+O(\epsilon^2)$. These values are well known [14,15]. At the critical dimension $d_c=2$ we proceed in the following manner. The observation time t is related to an initial time t_0 by

$$t=t_0\exp\left(\int_0^\xi z(\xi')d\xi'\right). \tag{25}$$

Using Eqs. (23),(24) we can fix the scaling parameter ξ according to Eq. (25) to be

$$\xi\simeq\frac{1}{2}\ln\left(\frac{t}{t_0}\right)+\frac{1}{2}\ln\left(1+\frac{a_0}{2}\frac{t}{t_0}\right),$$

where a_0 is the initial value for the parameter a. From Eq. (24) we find the following behavior for the mean square displacement:

$$\langle\mathbf{r}^2\rangle=c_1\frac{t}{t_0}+c_2\frac{t}{t_0}\ln\left(\frac{t}{t_0}\right), \tag{26}$$

where c_1 and c_2 are two nonuniversal constants. As expected, the system reveals logarithmic corrections at the critical dimension.

One should note that due to considering a short range force-force correlation function defined by Eq. (22), the critical dimension is 2 as has been discussed above. In the case of a short range energy-energy correlation function the critical dimension is shifted to $d_c=0$ because of $f\sim q\varepsilon$.

V. BEHAVIOR ABOVE THE CRITICAL DIMENSION

The thermalized version of the Fock-space representation [see Eqs. (8),(9)], leads in the limit $T\to\infty$ to conventional diffusion. In the high temperature limit the particles are able to overcome each barrier and as a consequence of the stochastic hopping process one finds diffusive behavior in the long time limit independently of the underlying statistics. When the temperature is finite there appears a competition between two processes resulting in a different behavior for both systems. Bose particles can easily find a minimum within the energy landscape defined by the stochastic force. Particles with exclusion have to search for a longer time and on a larger scale to reach an appropriate potential minimum. From here one would conclude an enhanced diffusivity. On the other hand, the mobility of bosons is eventually reduced because they find a stable minimum more rapidly. Due to the established universality for low dimensions, a variation in the behavior should be observed only above the critical dimension. In this regime conventional perturbation theory should be applicable. Let us therefore present lowest order corrections to the the diffusion parameter D. The effective diffusivity is defined by

$$D_{\text{eff}}=\left|\overline{\frac{\partial n^{-1}(\vec{q},\omega)}{\partial q^2}}\right|_{q=0,\omega=0}. \tag{27}$$

The Bose and the Fermi systems lead in second order, proportional to $1/T^2$, to nontrivial corrections which are also manifested in the averaged density $\overline{n(\mathbf{r},t)}$ or the averaged correlation function $\overline{n(\mathbf{r},t)n(\mathbf{r}',t')}$. Indeed, the Fermi system offers additional terms for the density or the correlation function compared with the Bose case. However, those terms do not contribute at zero wave vector and hence there are relevant corrections to the divergent part of D_{eff} for $d\leqslant d_c$. Above d_c the behavior of the effective diffusion coefficient can be estimated using a perturbative approach around the homogeneous solution denoted by $\overline{n}$. We get

$$D_{\mathrm{eff}}^{f}=D_{\mathrm{eff}}^{b}+\frac{(1-\overline{n})\overline{n}}{DT^{2}}I$$

with

$$I=\frac{4K_d}{d}I_1[B-A(d-1)]. \tag{28}$$

I_1 can be expressed by a momentum integral which is always positive in the mesoscopic regime $\Lambda>l$. For $B-A(d-1)>0$, realized for a pure potential field [B is the relevant variable, see Eq. (22)], Eq. (28) leads to

$$D_{\mathrm{eff}}^{f}>D_{\mathrm{eff}}^{b}. \tag{29}$$

We remark that the correction to the bare diffusion coefficient D is of the order $(1-2\overline{n})^2$, which means for the half-filled case there are no corrections. That reasonable result should also be valid in a more refined approach.

Because the homogeneous solution is not necessarily a stable one we can also estimate the behavior using linear stability analysis around the stationary solution denoted as $n_s(\mathbf{r})$. Let us introduce $n(\mathbf{r},t)=n_s(\mathbf{r})+y(\mathbf{r},t)$ then the correction $y(\mathbf{r},t)$ fulfills, in the Bose case, the equation

$$\partial_t y=D\boldsymbol{\nabla}^2 y+\frac{D}{T}\boldsymbol{\nabla}(y\boldsymbol{\nabla}\varepsilon_b) \quad \text{with } \mathbf{f}(\mathbf{r})=-\nabla\varepsilon_b(\mathbf{r}). \tag{30}$$

Here $\varepsilon_b(\mathbf{r})=\varepsilon(\mathbf{r})-v$ is the true stochastic potential introduced by Eq. (10) and v plays the role of the chemical potential which regulates the occupation number. In the case of the exclusion model the deviation from the stationary solution $y(\mathbf{r},t)$ satisfies the same equation, however, one has to replace the potential in the Bose case, given in Eq. (30), by another effective potential

$$\varepsilon_b(\mathbf{r})\rightarrow\varepsilon_f(\mathbf{r})=2T\ln\left[\frac{\cosh\{[\varepsilon(\mathbf{r})-v]/2T\}}{\cosh(v/2T)}\right]. \tag{31}$$

We have gauged the potentials so that for $\varepsilon_f(\mathbf{r})=0$ also $\varepsilon(\mathbf{r})=0$. The hopping particles under exclusion are subjected to the modified stochastic energy landscape given by ε_f. Expanding ε_f in terms of ε we find the relation

$$\varepsilon_f(\mathbf{r})\simeq-\tanh\left(\frac{v}{2T}\right)\varepsilon(\mathbf{r}). \tag{32}$$

From here we find

$$\overline{\varepsilon_f(\mathbf{r})\varepsilon_f(0)}\simeq\tanh^2\left(\frac{v}{2T}\right)\ \overline{\varepsilon_b(\mathbf{r})\varepsilon_b(0)}. \tag{33}$$

The effective correlator of the disorder in the Fermi system is drastically decreased in comparison to the Bose case. This result is compatible with the previous discussion leading to Eqs. (28),(29). In particular in the vicinity of half filling (where the chemical potential v is zero) the influence of the disorder is very weak. This special case corresponds to the vanishing linear term, expanding Eq. (31) according to powers of ε. To leading order we obtain

$$\varepsilon_f(\mathbf{r})\simeq\frac{\varepsilon^2(\mathbf{r})}{4T}.$$

Differently, from the Bose case, the effective stochastic potential ε_f, Eq. (31), is always positive definite, which means that all the deep negative minima of the original stochastic potential become maxima and therefore they are not more available in the case of the Fermi system. Obviously, they are already occupied and hence they are not accessible for particles.

It should be noted that our approach includes only a single approximation, namely, the decoupling of the nonlinear term in Eq. (12). Whenever the nonlinear term is irrelevant (as demonstrated in Sec. IV using renormalization group arguments) the approximation is reasonable. The mentioned nonlinear term becomes relevant for $d>d_c$ which is, for instance, manifested in calculating the stationary solution $n_s(\mathbf{r})$. Obviously, the higher the dimension is, the more accurate the approximation will be. For dimensions low but slightly above d_c we expect further corrections due to the fact that the nonlinear term considers percolation effects of occupied sites. In the case of a short range force-force correlation the main conclusion, Eq. (29), holds for all dimension $d>d_c=2$. The situation for low dimensions is changed in the case of a short range energy-energy correlation ($d_c=0$). In particular, the decoupling of higher order terms is no longer justified. Especially, the case $d=1$ remains an open problem.

VI. CONCLUSIONS

In the present paper the collective hopping process on a lattice is studied systematically when the particles are subjected to a random energetic landscape manifested by a stochastic energy profile. In particular, we have taken into account both cases, each lattice site being occupied by only one particle or each site absorbing an arbitrary number of particles. Physically, one expects a different behavior. Whereas in the situation under exclusion a particle should spend more time searching for an appropriate energy minimum within the stochastic energy, the bosons tend to reduce their mobility because they remain for a longer time in the local minima. A further influence on the motion of the particles is given by the coupling to a heat bath, which supports the tendency of the system to equilibrate. Starting on a master equation in a second quantized form, both cases can be easily realized in terms of Bose operators or spin-1/2 Pauli operators. The annihilation and creation process of particles leads in both cases to a density gradient characteristic of a random walk. Due to the additional coupling to stochastic energy, each particle cannot follow that gradient simply but has to overcome an energy barrier at its starting point and at its end point. A conflicting situation appears where a particle follows the density gradient but the energy at the starting point is higher than at the end point. In this manner it will jump from an occupied to an empty site, however, under mobilization of a higher amount of energy (lower temperature). The other situation consists of the fact that a particle follows the density gradient and the energy barrier at the starting point is lower than at the end point (high-temperature regime). In this case the hopping process is highly supported by the energy

landscape whereas in the previous one the process is restricted. As a consequence, the anomalous diffusive behavior should be realized below the critical dimension.

In this paper we have demonstrated that the Bose system—as well as the Fermi system—belongs below the critical dimension, to the same universality class within the long time limit and on a large spatial scale. For an increasing scale the system can be considered to consist of blocks with an increasing number of particles. Thus, the cases of restricted and unrestricted occupation number per lattice site should be irrelevant. Despite the universality the density and the correlation function of both systems are different, in particular for an intermediate interval. In particular, we have discussed the situation above the critical dimension where the diffusion constant can offer different behaviors in both cases.

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